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Dynamics of a rigid cylinder near a plane boundary in the radiation field of an acoustic wave

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Abstract

An approach is described for investigation of the interaction between a rigid body and a viscous fluid boundary under acoustic wave propagation. The influence of the liquid on the rigid body is determined as a mean force, which is a constant in the time component of the hydrodynamic force. This enables the use of a previously developed technique for calculation of pressure in a compressible viscous liquid. The technique takes into account the second-order terms with respect to the wave field parameters and is based on investigation of a system of initially nonlinear hydromechanics equations that can be simplified with respect to the wave motion parameters of the liquid. It has proven possible to retain the second-order terms for determination of stresses in the liquid without having to solve the system of nonlinear equations. The stresses can be expressed in terms of parameters found in the solution of the linearized equations of the compressible viscous liquid. In this way, the solution of linearized equations is expressed in terms of a scalar and vector potentials. The problem statement is derived for a rigid cylinder located near a rigid flat wall under the effects of a wave propagating perpendicular to the wall. The solution for this particular example is obtained.
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1. Introduction

In practice, many problems arise that involve the study of dynamic processes in fluids, and acoustic excitation is employed in many technological processes based on a fluid–particle system. Studies of particle dynamics are therefore of particular interest in this case. In industry, there exist technological processes (using particle coagulation in a fluid with subsequent sedimentation) in which time-averaged (radiation) forces play a determining role. Various aspects of the cylinder motion in liquid have been addressed in previous works (Morse et al., 2008; Pasto, 2008).

The study of particle motion under the influence of time-averaged forces (mean motion) is a complicated problem. More precise prediction of the solids' behaviour requires consideration of the finiteness of the space filled with the liquid, since the interaction process between the solids and the liquid is significantly affected by the presence of boundaries. Solid particles near a fluid boundary are in an interference field composed of primary and reflected waves, which determines the interaction of the boundary and particles via the medium. The wave interference field creates a time-averaged force whose magnitude and direction are functions of many factors: the angle of incidence of the wave on

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the boundary surface, the ratio of the wavelength to the distance between particle and boundary, the shape of the boundary, etc.

In this work, we consider a rigid body located near the interface between a compressible viscous fluid and a rigid flat wall. We define the mean forces as the mean (with respect to the wave field period) values of the integrated forces of the fluid acting on the body. In this case, the linear approximation for determination of these forces is insufficient. That approximation provides a periodic force that leads to a zero mean value. The mean force becomes evident as a result of taking into account the second-order effects and it has the same order of magnitude. This means that for stress calculations in the liquid, the nonlinear equation should be used and second-order terms with respect to the wave field parameters should be retained. Thermal effects in the liquid will be neglected in further analysis. In this case, the problem can be solved by the method proposed in Guz and Zhuk (1982, 1993, 2004) for an unbounded fluid. According to this method, the tensor field of stresses in the fluid is represented in terms of the potentials of the primary and secondary waves with accuracy to values of the order of the Mach number. The secondary-wave potentials are determined by solving a diffraction problem formulated on the basis of the linearized theory of a compressible viscous fluid (Guz, 1981, 1998, 2000a, b, 2009).

We consider a circular cylinder of radius a parallel to a flat rigid wall at distance δ. The principal coordinate system is such that its origin, point O, is in the plane of the wall (Fig. 1). The Ox_3 axis is parallel to the axis of the cylinder, and the Ox_1 axis is perpendicular to the plane of the wall so that it intersects the axis of the cylinder. It is assumed that a plane pressure wave is propagated in the negative direction of the Ox_1 axis. In accordance with this method, the mean force acting on the cylinder is filtered out by time-averaging the surface integral of the convolution of the stress tensor in

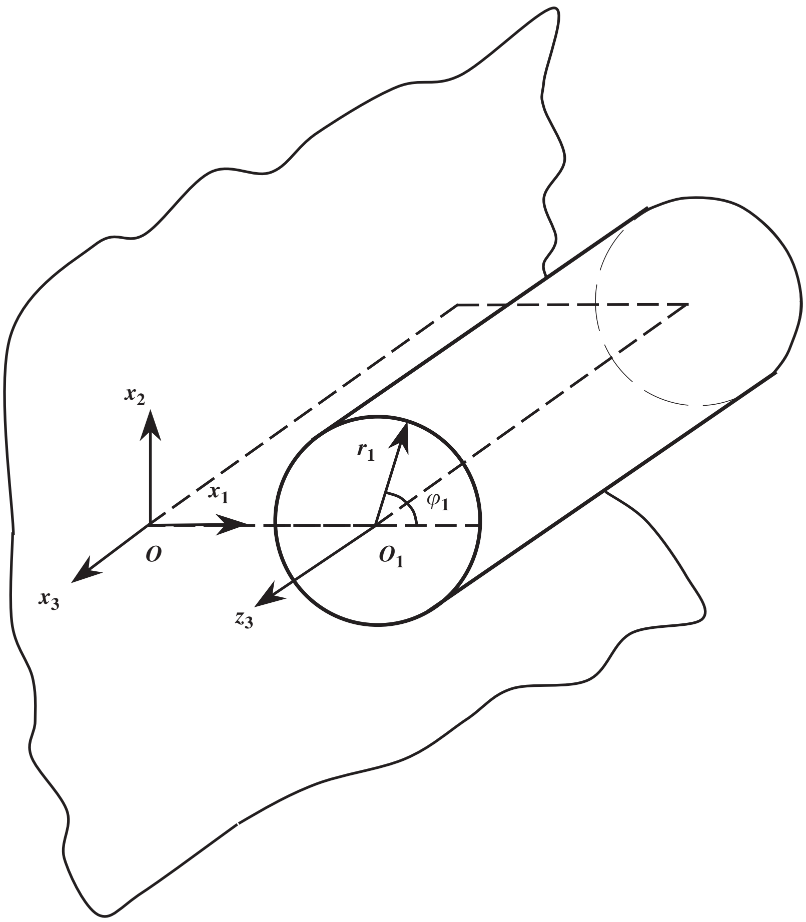

Fig. 1. Cylinder and rigid wall location.

the fluid with the basis vector of the normal to the surface S of the cylinder

$$\mathbf{F} = \iint_S \hat{\mathbf{\Sigma}} \cdot \mathbf{e}_r \mathrm{d}S, \tag{1}$$

where $\hat{\mathbf{\Sigma}}$ is the tensor of stresses in the fluid, and $\mathbf{e}_r$ is the basis vector of the normal to the lateral surface S of the cylinder, which we define on a unit length of the cylinder. In the calculation of the stress tensor

$$\hat{\mathbf{\Sigma}} = (-p + \lambda' \nabla \cdot \mathbf{v})\hat{\mathbf{E}} + 2\mu' \hat{\mathbf{e}} \tag{2}$$

the pressure in the fluid must be (Guz and Zhuk, 1982)

$$p = \rho_0 \left(\frac{\lambda' + 2\mu'}{\rho_0} \Delta - \frac{\partial}{\partial t} \right) \Phi + \frac{\rho_0}{2a_0^2} \left(\frac{\partial \Phi}{\partial t} \right)^2 - \frac{1}{2} \rho_0 (\nabla \Phi)^2 - \frac{\lambda' + 2\mu'}{a_0^2} \frac{\partial \Phi}{\partial t} \Delta \Phi. \tag{3}$$

In Eqs. (2) and (3), ρ_0 and a_0 are the density of the sound velocity in the motionless fluid, $\mathbf{v}$ is the velocity vector of particles of the fluid, p is the pressure perturbation, λ' and $\mu'$are the dynamic and second viscosity coefficients, $2\hat{\mathbf{e}} = \nabla \mathbf{v} + (\nabla \mathbf{v})^{\mathrm{T}}$ is the deformation-velocity tensor, and $\hat{\mathbf{E}}$ is a unit tensor. The scalar potential Φ of the wave field is obtained from the solution of the linear problem of the incident and reflected waves for the cylinder. The partial case of the ideal compressible liquid (King, 1934) can be obtained by the limiting transition for λ', $\mu' \to 0$ in expression (3).

2. Procedure for velocity field potentials obtained from solution of the diffraction problem

It is assumed that the velocity potential of the incident wave is given by the expression

$$\Phi_i = A \exp[-\mathrm{i}(\gamma x_1 + \omega t)], \tag{4}$$

where A is the wave amplitude for $x_1 = 0$, $\gamma = k + \mathrm{i}\xi_1$ is the complex-value wavenumber, and ω is the circular frequency.

From a mathematical point of view, the diffraction problem for an incident wave on the cylinder can be formulated on the basis of the linearized theory of compressible viscous liquid (Guz, 1981, 1998). Solution of the problem is reduced to the solution of equations

$$\left[\left(1 + \frac{\lambda' + 2\mu'}{a_0^2 \rho_0} \frac{\partial}{\partial t} \right) \Delta - \frac{1}{a_0^2} \frac{\partial^2}{\partial t^2} \right] \Phi = 0, \tag{5}$$

$$\left(\frac{\mu'}{\rho_0} \Delta - \frac{\partial}{\partial t} \right) \mathbf{\Psi} = 0. \tag{6}$$

The velocity field $\mathbf{v}$ in the liquid can be obtained as follows:

$$\mathbf{v} = \nabla \Phi + \nabla \times \mathbf{\Psi}, \quad \nabla \cdot \mathbf{\Psi} = 0 \tag{7}$$

and should satisfy next three conditions:

(i) boundary condition on the wall surface

$$\mathbf{v} = 0, \quad \text{for } x_1 = 0; \tag{8}$$

(ii) boundary condition on the lateral surface of the cylinder

$$\mathbf{v} = \mathbf{V}; \tag{9}$$

(iii) fade condition at infinity of the wave reflected by the cylinder.

The velocity vector $\mathbf{V}$ of the cylinder, provided that it is cylindrically isotropic, is determined from the equation of motion in the fluid

$$m\dot{\mathbf{V}} = \iint_S \hat{\boldsymbol{\sigma}} \cdot \mathbf{e}_r \mathrm{d}S, \tag{10}$$

in which m is the mass of a unit length of the cylinder. In the linearized theory, the stress tensor $\hat{\boldsymbol{\sigma}}$ is calculated by the formula

$$\hat{\boldsymbol{\sigma}} = (-p' + \lambda' \nabla \cdot \mathbf{v})\hat{\mathbf{E}} + 2\mu' \hat{\mathbf{e}}, \tag{11}$$

where

$$p' = \rho_0 \left(\frac{\lambda' + 2\mu'}{\rho_0} \Delta - \frac{\partial}{\partial t} \right) \Phi. \tag{12}$$

Formula (7) is used to determine the components of the deformation-velocity tensor $\hat{\mathbf{e}}$.

For the problem at hand, wave (4), reflected from the flat surface of the rigid wall, will not be converted to a shear wave; therefore, the potential of the reflected wave can be represented as

$$\Phi_1 = A \exp[\mathrm{i}(\gamma x_1 - \omega t)]. \tag{13}$$

Then, the problem formulated above is reduced to the problem of diffraction of waves (4) and (13) on the cylinder located near the plane wall. In order to solve the problem, we use the approaches developed in the book by Guz and Golovchan (1972) for problems of elastic-wave diffraction in multiply connected bodies. We construct solutions of Eqs. (5) and (6) by separation of variables in cylindrical coordinate systems. For this, we relate the cylinder to a local cylindrical coordinate system $O_1 r_1 \phi_1 z_3$ (see Fig. 1) in which the coordinate origin is the point of intersection of the cylinder axis with the Ox_1 axis and the Oz_3 axis directed along the cylinder's axis. In the case in question, the vector potential $\boldsymbol{\Psi}$ is determined via a single scalar $\boldsymbol{\Psi}_1$ (Guz and Golovchan, 1972)

$$\boldsymbol{\Psi} = \mathbf{e}_1 \Psi_1. \tag{14}$$

Using Jacobi expansions, we represent the expressions for plane waves (4) and (13) in the cylindrical coordinate system $O_1 r_1 \phi_1 z_3$ in terms of the cylindrical wave functions

$$\Phi_i = A \mathrm{e}^{-\mathrm{i}\gamma\delta} \sum_{n=0}^{\infty} (-1)^n e_n \mathrm{i}^n \mathrm{J}_n(\gamma r_1) \cos(n\phi_1), \tag{15}$$

$$\Phi_1 = A \mathrm{e}^{\mathrm{i}\gamma\delta} \sum_{n=0}^{\infty} e_n \mathrm{i}^n \mathrm{J}_n(\gamma r_1) \cos(n\phi_1), \tag{16}$$

where $e_0 = 1$, $e_n = 2$ $(n \geq 1)$, and $\mathrm{J}_n(z)$ are Bessel functions of argument z.

The velocity field of the wave reflected by the cylinder should satisfy boundary condition (8) on the surface $x_1 = 0$. This condition is satisfied in the cylindrical coordinate system if an imaginary representation technique is employed. Let us assume that the entire space is filled with liquid and there is a second cylinder located symmetrically to the first with respect to the plane $x_1 = 0$. It is sufficient to show that the velocity field produced by the waves reflected by the cylinders complies with condition (8). Coordinate system $O_2 r_2 \phi_2 z_3'$ related to the second cylinder is introduced. Then, solutions of Eqs. (5) and (6) (i.e., potentials of pressure waves $\Phi_d^{(s)}$ and shear waves $\Psi_1^{(s)}$ reflected by the cylinders, where $s = 1{,}2$ is the cylinder number) are expressed in terms of generalized Fourier series

$$\Phi_d^{(s)} = \sum_{n=0}^{\infty} A_n^{(s)} \mathrm{H}_n^{(1)}(\gamma r_s) \cos(n\phi), \tag{17}$$

$$\Psi_1^{(s)} = \sum_{n=1}^{\infty} B_n^{(s)} \mathrm{H}_n^{(1)}(\beta r_s) \sin(n\phi), \tag{18}$$

where $\beta = \sqrt{\mathrm{i}\omega\rho_0/\mu'}$ is the wavenumber of the shear wave, and $A_n^{(s)}$ and $B_n^{(s)}$ $(s = 1{,}2)$ are the integration constants.

Using formulas (7), (17), and (18), we find that condition (8) for the waves reflected from the cylinders establishes the following relationship between the coefficients in series (17) and (18):

$$A_n^{(2)} = (-1)^{n+1} A_n^{(1)}, \quad B_n^{(2)} = (-1)^{n+1} B_n^{(1)}.$$

Constants $A_n^{(1)}$ and $B_n^{(1)}$ can be found from boundary condition (9) on the surface of the first cylinder. To this end, it is necessary to rewrite the potentials of the velocity field (7)

$$\Phi = \Phi_i + \Phi_1 + \Phi_d^{(1)} + \Phi_d^{(2)}, \quad \Psi_1 = \Psi_1^{(1)} + \Psi_1^{(2)}$$

in terms of coordinate system $O_1 r_1 \phi_1 z_3$ related to the first cylinder.

Employing the addition theorems for the cylindrical wave functions (e.g. see Guz and Golovchan, 1972), the following expressions can be deduced:

$$\Phi_d = \sum_{s=1}^{2} \Phi_d^{(s)} = \sum_{n=0}^{\infty} [A_n^{(1)} \mathrm{H}_n^{(1)}(\gamma r_1) + e_n S_n^{(1)} \mathrm{J}_n(\gamma r_1)] \cos(n\varphi_1), \tag{19}$$

$$\Psi_1 = \sum_{s=1}^{2} \Psi_1^{(s)} = \sum_{n=1}^{\infty} [B_n^{(1)} \mathrm{H}_n^{(1)}(\beta r_1) + e_n Q_n^{(1)} \mathrm{J}_n(\beta r_1)] \sin(n\varphi_1),$$

$$S_n^{(1)} = \frac{1}{2} \sum_{m=0}^{\infty} (-1)^{m+1} \alpha_{2nm} A_m^{(1)}, \quad Q_n^{(1)} = \frac{1}{2} \sum_{m=0}^{\infty} (-1)^{m+1} \alpha_{1nm} B_m^{(1)},$$

$$\alpha_{1nm} = \mathrm{H}_{n-m}^{(1)}(2\beta\delta) - (-1)^m \mathrm{H}_{n+m}^{(1)}(2\beta\delta),$$

$$\alpha_{2nm} = \mathrm{H}_{n-m}^{(1)}(2\gamma\delta) + (-1)^m \mathrm{H}_{n+m}^{(1)}(2\gamma\delta). \tag{20}$$

From the condition of symmetry of the wave field with respect to the plane Ox_1x_3, it follows that the cylinder will move along the Ox_1 axis under the influence of an acoustic wave. Writing the right side of (10) in the cylindrical coordinate system and taking (11) and (12) into account, after integration we obtain the following formula for the projection of the cylinder velocity onto the Ox_1 axis:

$$V_{X_1} = \frac{\eta}{a} [-2e_1 A \sin(\gamma\delta) \mathrm{J}_1(\gamma a) + A_1^{(1)} \mathrm{H}_1^{(1)}(\gamma a) + e_1 S_1^{(1)} \mathrm{J}_1(\gamma a) + B_1^{(1)} \mathrm{H}_1^{(1)}(\beta a) + e_1 Q_1^{(1)} \mathrm{J}_1(\beta a)]. \tag{21}$$

The factor $\exp(-\mathrm{i}\omega t)$ is omitted in formulas (15)–(21).

An infinite system of algebraic equations for constants $A_n^{(1)}$ and $B_n^{(1)}$ is derived from boundary condition (9), taking into account (15), (16), (19)–(21), and (7) (not shown here due to complexity). The system has a unique solution that can be found by a reduction method. The desired accuracy is ensured by comparing the calculation results for a successively increasing number of terms. Calculation of values for constants $A_n^{(1)}$ and $B_n^{(1)}$ completes the determination procedure for the velocity field potentials Φ and $\mathbf{\Psi} = \mathbf{e}_3 \Psi_1$.

3. Calculation of time-averaged force

By making use of velocity field potentials deduced from the linear equations, we can determine the pressure (3) and stress (2) in the liquid with accuracy to the second-order term. Thus, a hydrodynamic force acting on the cylinder can be found with the same accuracy. In view of the symmetry of the wave field, the force will be directed along the Ox_1 axis. Using expressions (7), (3), and (2), the component of hydrodynamic force can be deduced from Eq. (1):

$$F_{x_1} = \iint_S \mathbf{i}_1 \cdot \hat{\mathbf{\Sigma}} \cdot \mathbf{e}_{r_1} \mathrm{d}S. \tag{22}$$

Having averaged (22) over time, the corresponding time-averaged force can be determined:

$$\langle F_{x_1} \rangle = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} F_{x_1} \mathrm{d}t. \tag{23}$$

Applying the approaches used in Zhuk (1991), a numerical solution for time-averaged force can be calculated in the particular case of the cylinder located near the plane wall, and motion of the body in the liquid caused by the force can be determined.

The approach considered here can be used to solve similar problems in the case of a soft boundary or in the case of a flat interface between two fluids.

4. Example

As an example, let us consider the motion of a cylinder caused by the time-averaged radiation force in an ideal liquid near the plane solid wall. The acoustic wave propagates perpendicular to the plane of wall such that

$$\Phi_i = A \exp[-\mathrm{i}(kx_1 + \omega t)]. \tag{24}$$

The main expressions for the case of an ideal liquid can be obtained from those deduced above by means of the limiting conditions $\lambda' \to 0$ and $\mu' \to 0$. If we constrain this case with additional conditions $ka \gg 1$ and $k\delta \gg 1$, the velocity field (7) becomes a superposition of the incident wave and the wave reflected by the wall. In this case, the time-averaged radiation force (23) and the velocity field can be expressed as follows [e.g., see Guz et al. (1994)]:

$$\langle F_{X_1} \rangle = A^2 \pi \rho_0 \sin(2k\delta) \left[\frac{3-\eta}{a(1+\eta)} \right] (ka)^3, \tag{25}$$

$$v_{X_1} = -2Ak \sin(kx_1) \cos(\omega t), \tag{26}$$

where η is a ratio of the density of liquid ρ_0 to the density of the cylinder ρ_1.

Let us investigate the movement for a cylinder initially located between the loop and the node. Planes $x_1 = n\pi/k$ ($n = 0, 1, \ldots$) are the modes and planes$x_1 = (n + \frac{1}{2})\pi/k$ ($n = 0, 1, \ldots$) are the loops of the velocity field (26). The sign of the product of the terms $\sin(2k\delta)$ and $(3-\eta)$ in (25) determines the direction of the time-averaged force action. Analysis of expression (25) shows that the time-averaged force shifts the more dense cylinder ($\eta < 3$) to the loop, and the lighter cylinder ($\eta > 3$) moves to the node. The equation of motion in the latter case can be written as follows:

$$(\pi a^2 \rho_1 + \pi a^2 \rho_0)\ddot{\delta} = \pi \rho_0 A^2 \sin(2k\delta) \left[\frac{3-\eta}{a(1+\eta)} \right] (ka)^3, \tag{27}$$

where $\pi a^2 \rho_0$ is the apparent additional mass per unit length of the cylinder.

The loops and nodes of the velocity field correspond to the equilibrium states for the cylinder. If we check the stability conditions for them, it can be shown that the mechanical system described by Eq. (27) is conservative (there exists a first integral over the entire phase plane). Substitution of $2k\delta = \pi - \theta + 2n\pi$ into Eq. (27) gives

$$\ddot{\theta} + \chi^2 \sin\theta = 0, \tag{28}$$

where

$$\chi^2 = 2A^2 k^4 \left[\frac{\eta(3-\eta)}{(1+\eta)^2} \right].$$

Eq. (28) does not depend on the cylinder radius and is the equation of free vibrations of the nonlinear oscillator. Therefore, the cylinder accomplishes vibrational motion with respect to the position of stable equilibrium (Magnus, 1976).

The points of stable equilibrium for $\chi^2 > 0$ (relatively dense cylinder) are determined by the zero values of θ (loops of the velocity field). These points are the loops of the velocity field: $\delta = (n + \frac{1}{2})\pi/k$ ($n = 0, 1, \ldots$). In this case the nodes of the velocity field are the unstable equilibrium points. The period of the cylinder vibration is equal to

$$T = \frac{\lambda}{v_a} \frac{1+\eta}{\sqrt{2\eta(3-\eta)}} \frac{2}{\pi} \mathrm{K}(\varepsilon), \tag{29}$$

where $\mathrm{K}(\varepsilon)$ is the total elliptic integral of the first kind and λ is the wavelength; v_a is the peak velocity of the particles of the liquid, $\varepsilon = \sin(kx_0)$, with x_0 indicating the maximum deflection of the cylinder from the loop in its mean motion. The vibration period (29) has a minimum at $\eta = 0.6$:

$$T_{\min} = 0.943 \frac{\lambda}{v_a} \frac{2}{\pi} \mathrm{K}(\varepsilon). \tag{30}$$

The points of stable equilibrium for $\chi^2 < 0$ (relatively light cylinder) are determined by the values $\theta = \pi$ (nodes of the velocity field: $\delta = n\pi/k$, $n = 0, 1, \ldots$). In this case, the loops of the velocity field are the points of unstable equilibrium. The period of vibration is equal to

$$T = \frac{\lambda}{v_a} \frac{1+\eta}{\sqrt{2\eta(\eta-3)}} \frac{2}{\pi} \mathrm{K}(\varepsilon) \tag{31}$$

and does not have any extremes. The notations used here is the same as in formula (30), except that x_0 denotes maximum deflection from the node.

The same technique can be used to solve this problem for a spherical particle (Zhuk, 2008).

5. Conclusions

A cylinder located in the liquid near the solid wall undergoes vibrational motion under the influence of radiation force from the acoustic field; it vibrates with respect to the position of stable equilibrium, depending on both the wavenumber of the acoustic wave and the ratio between the liquid density and the density of the cylinder. Analysis of formulas (30) and (31) shows clearly that the vibration period of the cylinder in motion under the influence of radiation force is dependent on wavelength, velocity amplitude of liquid particles, ratio of densities, and the initial location of the cylinder with respect to the stable equilibrium point. The radius of the cylinder has no effect on the period of vibration.

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